UNIT – 3 (TRANSPORTATION PROBLEM)

The transportation problem is a special class of the linear programming problem. It deals with the situation in which a commodity is transported from ***Sources*** to ***Destinations***. The objective is to determine the amount of commodity to be transported from each source to each destination so that the total transportation cost is minimum.

EXAMPLE:-

A soft drink manufacturing firm has *m* plants located in ***m*** different cities. The total production is absorbed by ***n*** retail shops in n different cities. We want to determine the transportation schedule that minimizes the total cost of transporting soft drinks from various plants to various retail shops. First we will formulate this as a linear programming problem.

MATHEMATICAL FORMULATION

Let us consider the m-plant locations (origins) as O_1 , O_2 , …., O_m and the n-retail shops (destination) as D_1 , D_2 , ….., D_n respectively. Let $a_i >= 0$, $i= 1,2, \ldots m$, be the amount available at the i^{th} plant O_i . Let the amount required at the j^{th} shop D_j be $b_j >= 0, j= 1,2,\ldots n.$ Let the cost of transporting one unit of soft drink form ith origin to *jth* destination be C_{ij} , $i= 1,2, \ldots m$, $j=1,2,\ldots n$. If $x_{ij} >= 0$ be the amount of soft drink to be transported from i^{th} origin to *jth* destination , then the problem is to determine x_{ij} so as to Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} c_{ij}$$

Subject to the constraint

and $x_{ij} \geq 0$, for all i and j.

$$\sum_{j=1}^{n} x_{ij} = a_i , i = 1,2,\ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j , j = 1,2,\ldots n.$$

This LPP is called a ***Transportation Problem.***

THEOREM:

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Remark. The set of constraints

$$\sum_{i=1}^{m} x_{ij} = b_j \; and \; \sum_{j=1}^{n} x_{ij} = a_i$$

Represents *m+n equations* in *mn* non-negative variables. Each variable xij appears in exactly two constraints, one is associated with the origin and the other is associated with the destination.

Note. If we are putting in the matrix from, the elements of A are either 0 or 1.

THE TRANSPORTATYION TABLE:

Source\Destination	**D_1**	**D_2**			**D_n**	**Supply**
O_1	**C_{11}**	**C_{12}**	**…..**	**…..**	**C_{1n}**	**a_1**
O_2	**C_{21}**	**C_{22}**	**…..**	**…..**	**C_{2n}**	**a_2**
.	**…..**	**…..**	**…..**	**…..**	**…..**	**…**
	…..	**…..**	**…..**	**…..**	**…..**	**---**
O_m	**C_{m1}**	**C_{m2}**	**…..**	**…..**	**C_{mn}**	**a_m**
Requirements	**b_1**	**b_2**	**…..**	**…..**	**b_n**	

Definition. (Loop). In a transportation table, an ordered set of four or more cells is said to form a loop if :

(I) Any two adjacent cells in the ordered set lie in the same row or in the same column.

(II) Any three or more adjacent cells in the ordered set do not lie in the same row or in the same column.

Formulating Transportation Problems

Example 1: Powerco has three electric power plants that supply the electric needs of four cities. The associated supply of each plant and demand of each city is given in the table 1. The cost of sending 1 million kwh of electricity from a plant to a city depends on the distance the electricity must travel.

Transportation tableau

A transportation problem is specified by the supply, the demand, and the shipping costs. So the relevant data can be summarized in a transportation tableau. The transportation tableau implicitly expresses the supply and demand constraints and the shipping cost between each demand and supply point.

Table 1. Shipping costs, Supply, and Demand for Powerco Example

From	To				
	City 1	City 2	City 3	City 4	Supply (Million kwh)
Plant 1	$8	$6	$10	$9	35
Plant 2	$9	$12	$13	$7	50
Plant 3	$14	$9	$16	$5	40
Demand (Million kwh)	45	20	30	30	

Solution

1. **Decision Variable:**
   Since we have to determine how much electricity is sent from each plant to each city;
   X_{ij} = Amount of electricity produced at plant i and sent to city j
   X_{14} = Amount of electricity produced at plant 1 and sent to city 4
2. **Objective function:**
   Since we want to minimize the total cost of shipping from plants to cities;
   Minimize $Z = 8X_{11}+6X_{12}+10X_{13}+9X_{14}$
   $+9X_{21}+12X_{22}+13X_{23}+7X_{24}$
   $+14X_{31}+9X_{32}+16X_{33}+5X_{34}$
3. **Supply Constraints**
   Since each supply point has a limited production capacity;
   $X_{11}+X_{12}+X_{13}+X_{14} <= 35$
   $X_{21}+X_{22}+X_{23}+X_{24} <= 50$
   $X_{31}+X_{32}+X_{33}+X_{34} <= 40$
4. **Demand Constraints**
   Since each supply point has a limited production capacity;
   $X_{11}+X_{21}+X_{31} >= 45$
   $X_{12}+X_{22}+X_{32} >= 20$
   $X_{13}+X_{23}+X_{33} >= 30$
   $X_{14}+X_{24}+X_{34} >= 30$
5. **Sign Constraints**
   Since a negative amount of electricity can not be shipped all Xij's must be non negative;
   Xij >= 0 (i= 1,2,3; j= 1,2,3,4)

LP Formulation of Powerco's Problem

Min $Z = 8X_{11}+6X_{12}+10X_{13}+9X_{14}+9X_{21}+12X_{22}+13X_{23}+7X_{24} +14X_{31}+9X_{32}+16X_{33}+5X_{34}$

S.T.: $X_{11}+X_{12}+X_{13}+X_{14} <= 35$ (Supply Constraints)
$X_{21}+X_{22}+X_{23}+X_{24} <= 50$
$X_{31}+X_{32}+X_{33}+X_{34} <= 40$
$X_{11}+X_{21}+X_{31} >= 45$ (Demand Constraints)
$X_{12}+X_{22}+X_{32} >= 20$

$X_{13}+X_{23}+X_{33} >= 30$
$X_{14}+X_{24}+X_{34} >= 30$

$Xij >= 0$ (i= 1,2,3; j= 1,2,3,4)

General Description of a Transportation Problem

1. A set of *m supply points* from which a good is shipped. Supply point *i* can supply at most s_i units.
2. A set of *n demand points* to which the good is shipped. Demand point *j* must receive at least d_i units of the shipped good.
3. Each unit produced at supply point *i* and shipped to demand point *j* incurs a *variable cost* of c_{ij}.

X_{ij} = number of units shipped from *supply point i* to *demand point j*

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} X_{ij}$$

$$s.t. \sum_{j=1}^{j=n} X_{ij} \le s_i (i = 1,2,\ldots,m)$$

$$\sum_{i=1}^{i=m} X_{ij} \ge d_j (j = 1,2,\ldots,n)$$

$$X_{ij} \ge 0 (i = 1,2,\ldots,m; j = 1,2,\ldots,n)$$

Balanced Transportation Problem: If Total supply equals to total demand, the problem is said to be a balanced transportation problem:

$$\sum_{i=1}^{i=m} s_i = \sum_{j=1}^{j=n} d_j$$

Balancing a TP if total supply exceeds total demand:
If total supply exceeds total demand, we can balance the problem by adding dummy demand point. Since shipments to the dummy demand point are not real, they are assigned a cost of zero.
Balancing a transportation problem if total supply is less than total demand:
If a transportation problem has a total supply that is strictly less than total demand the problem has no feasible solution. There is no doubt that in such a case one or more of the demand will be left unmet. Generally in such situations a penalty cost is often associated with unmet demand and as one can guess this time the total penalty cost is desired to be minimum

Finding Basic Feasible Solution for TP
Unlike other Linear Programming problems, a *balanced* TP with m supply points and n demand points is easier to solve, although it has m + n equality constraints. The reason for that is, if a set of decision variables (x_{ij}'s) satisfy all but one constraint, the values for x_{ij}'s will satisfy that remaining constraint automatically.

Methods to find the basic feasible solution (bfs) for a balanced TP:
There are three basic methods:

1. Northwest Corner Method
2. Minimum Cost Method
3. Vogel's Method

to be continued ………..